Ice Pack (Solution)

– Alex Walker, Eshan Mitra, Chris Yu –

The map of Antarctica shows 490 voting members: 144 penguins and 346 scientists. To win control of the parliament, a party must win at least 8 of the 14 districts. Since the narrowest margin in each district is 18-17, the minimal number of votes required to win is $8 \times 18 = 144$. In other words, a penguin victory *might* be possible, but only if the district map has been gerrymandered in a particular way.

To solve this puzzle, we must first determine the redistricting plan. While there are many ways to secure a scientist win, it turns out that there is only one way to secure a penguin win. Once this subdivision is found, we still need to extract an answer. Converting the 14 district areas into letters (via A=1,...,Z=26) and reading these letters in page order gives the answer **FRANK UNDERWOOD**.

For completeness, one path through the logic puzzle (and the resulting district map) is provided below. Steps which determine specific districts have been color-coordinated to match.

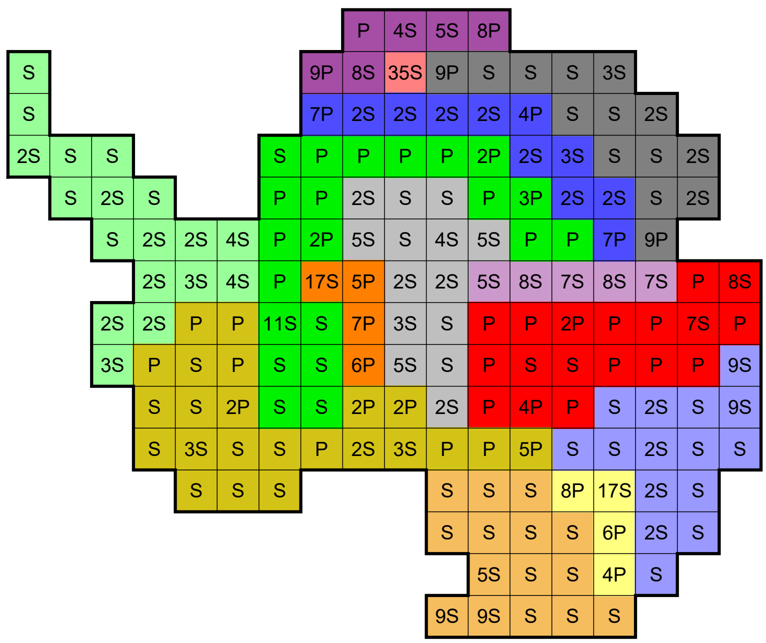
1. The single cell with 35S forms a complete district under scientist control.

2. The two 9S cells in the bottom row belong to the same district, which contains at least 18 scientists. We need 6 all-scientist districts and 8 districts with an 18-17 penguin majority, so this district must be the former. There is a unique way to add 17 more scientists without adding any penguins.

3. If the 17S cell near the bottom belongs to an all-scientist region, we need 18 more scientists in the region, with no additional penguins. This cell borders a region of 35 additional scientists, surrounded on all sides by penguins. Of these 35, 18 appear in the two 9S cells. Thus any district with 35 scientists must include at least one of the 9S cells. Yet any district which connects the 17S and 9S cells strands the 6P and 4P cells below in a way that can't form a full district. Thus the 17S cell belongs to a penguin-controlled district, and there is a unique way to add 18P to pull this off.

4. Any district containing the tip of the peninsula contains at least 13S, from the tip of the peninsula down to the second 2S choke point. If this belongs to a penguin district, it needs an extra 4S and 18P, which can't occur. We can form an all-scientist region containing the peninsula in a unique way.

5. Which other regions of the map support all-scientist districts? Several are plausible: the bottom left, the center (which supports two such regions), the bottom right, and the top right region. Three of these five will be used.



6. There is a large contiguous blob of 42 scientists near the top of the map. Any 35S district there must include the 8S cell and the 1S cell three cells to its right. This strands the 4S and 5S in the top row and gives a contradiction.

7. The large blob of scientists in the lower left consists of 45 scientists. To form an all-scientist district, we must include the 17S and 11S cells, leaving 7S to fill. Any attempt to add 7S subdivides the map leaving too few cells in the lower left to form a full district. Thus the last three all-scientist regions appear in the bottom right and in the center. The choice in the bottom right is forced.

8. The blob of scientists near the center of the map has exactly 70S. Starting at the rightmost cell, we add cells until we hit 35S (uniquely).

9. The other scientists in the center form the last all-scientist district.

10. Consider the region below the region from step 8. Between the right edge and the choke point near the center (under the 2S), we accrue 25P, which is 7P too many. There is a unique way to remove 7P without stranding cells. This naturally includes the desired 17S.

11. The central 17S can only connect to penguin cells. One option is to include the nearby 5P, 7P, and 6P cells. Otherwise, we must supplement with additional P cells farther up on the map, which occupies a choke point and separates the remainder of the lower left from the rest of the unaccounted-for cells. The remaining lower left would contain 33S, which leads to contradiction. Thus the 17S must connect to the 5P, 7P, and 6P cells.

12. The P to the left of the 17S from step 11 now acts as a choke point. There are 18P available below this choke point, which must belong together. There's a unique way to add on 17S without stranding anything.

13. The region containing the 11S includes all cells below and including the P choke point which lies three cells above the 17S from step 11. Once these cells are accounted for, we have 7P and 17S included, so we need 11P to finish the district. The nearby 7P and 9P cells don't help, so we head right and succeed in a unique way.

14. The remaining penguin cells have numbers 1, 4, 7, 7, 8, 9, 9, 9. The only way to sum to 18 using the 4 is 4+7+7. The minimal region connecting the 4P, 7P, and 7P (and not stranding cells) already uses 17S.

15. The region containing the 9P at top left must include the 8S, P, 4S, and 5S cells, to avoid stranding them. This forms a region with 10P and 17S, so must finish by including the 8P cell.

16. The remaining cells form the final region.

Construction Notes:

We wanted FRANK UNDERWOOD's puzzle to evoke underhanded politics, and settled on the idea of a gerrymandering logic puzzle early on in development. The hardest part of construction was forcing uniqueness – in a random map, regions can often swap pairs of cells near their border. We devised a variety of counter-measures:

- The original concept used districts with population 25. By increasing this to 35, we increased the per-cell populations on average, which is more forcing. (If too many cells are just 1S/1P, uniqueness is impossible.)
- The shape of the map (Antarctica) was chosen originally because the peninsula created a good choke point that could be used to easily define one of the districts. (The original plan was a 13×13 square.) The improved flavor was just a nice bonus.
- The all-scientist regions are particularly forcing and were placed strategically to either create walls or fill out large areas. (Smaller regions are easier to force, in that the per-cell populations are higher on average.)